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Dual Trading: Winners, Losers, and Market Impact*

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I show that dual trading reduces the net order flow and market depth. Trading volume and gross (of commission fees) profits of informed traders are lower with dual trading, while trading volume and gross losses of uninformed traders are unaffected. When the broker's commission income is independent of the customer's trading volume, the competitive commission fee is lower with dual trading. The utility of uninformed traders (net of commission fees) increases with dual trading, while the net profits of informed traders decrease. *Journal of Economic Literature* Classification Numbers: G12, G13, D82. © 1995 Academic Press, Inc.

Dual trading, the practice of brokers trading for their own accounts in addition to bringing their customers' orders to the market, is controversial. Proponents point to its salutary effects on market liquidity and price efficiency, whereas opponents emphasize the potential conflicts of interest. On the regulatory front, the anti-dual-trading camp currently holds sway. In March 1993, the Commodities Futures Trading Commission (CFTC) proposed limiting dual trading in the largest futures markets. The Chicago Mercantile Exchange (CME) banned dual trading in all "mature

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liquid" contracts effective May 20, 1991.[1] But, the issue remains unsettled.[2]

In what follows I develop a model for studying how dual trading affects market liquidity and price efficiency. I also examine distributional effects by looking at the impact on brokers' commission fees as well as on trading volumes and profits (both gross and net of commission costs) of informed and uninformed traders. The main contribution of this paper is to highlight the effect of dual trading on trading volume. In particular, I show that the trading volume of informed traders is reduced because the broker mimics or "piggybacks" on the informed trades.[3] Because their orders are now executed at a higher absolute price, the broker's piggybacking makes it costly for informed traders to buy and sell the amounts they would have chosen to trade in the absence of dual trading.[4]

Dual trading reduces market depth because the broker, in addition to mimicking informed trades, also offsets a portion of the uninformed order flow. This decreases the net order flow and increases the adverse selection problem for the marketmaker (who is assumed to observe only the net order flow). Yet, the *uninformed trading volume is a higher proportion* of the total expected trading volume with dual trading. In fact, the total expected trading volume is unaffected by dual trading as the loss in informed trading volume is offset by the broker's own trading activity. Further, price informativeness does not change with dual trading because market depth and the net order flow decrease in the same proportion.[5]

[1] One guideline would prohibit dual trading of "contracts with daily average volume of 10,000 contracts or more . . . over the previous six months" (CME Special Executive Report, May 3, 1991). As of May 20, 1991, the following contracts fell under the ban: first contract months of the Pound Sterling, Japanese Yen, Swiss Franc, Deutsche Mark, Eurodollar Options futures, and the first and second contract months of the Eurodollar futures contract.

[2] See *The Chicago Tribune*, February 4, 1992, and *The Wall Street Journal*, February 7, 1992, for details.

[3] The empirical evidence is mixed. Park *et al.* (1994) find that order imbalance of dual traders and customers in the S&P 500 and Japanese yen futures markets are positively correlated. However, Chang and Locke (1992) find no evidence of piggybacking in the currency futures markets.

[4] This is partially due to the batch market where the price is increasing in the net order flow, as in Kyle (1985). This characteristic of the model maximizes the negative impact of piggybacking on informed trading. The effect probably would remain in a multiperiod setting where customer and broker orders are executed and priced separately, so long as some subset of informed customers make repeat purchases of sales via the same broker. From Kyle (1985), the optimal dynamic trading strategy of an informed trader is to spread trades in small quantities over time.

[5] Empirical studies of dual trading effects on market depth and the bid–ask spread have produced mixed results. Many (Chang and Locke, 1992; Park and Sarkar, 1994; CFTC, 1990) find that dual trading does not significantly affect liquidity as measured by either the

Initially, commission fees are ignored. The model is an extension of Kyle (1985). Uninformed noise traders and a group of informed traders submit market orders to a broker who places them (along with her own orders) with a marketmaker for execution. The broker's motive for dual trading comes from her private observations of the size of her customers' orders. In equilibrium, she infers all that her informed customers know by piggybacking on their trades. The marketmaker batches the total order flow and executes at a single price. The price is determined by assuming that the marketmaker earns zero profits, conditional on observing the total order flow.

For the model without commission fees, the main result is that the profits of informed traders are reduced with dual trading, while uninformed traders neither gain nor lose. This result is unchanged when the model is extended to allow for rational behavior by uninformed traders. Following Spiegel and Subrahmanyam (1992), uninformed traders (who are risk-averse) trade in order to hedge their endowments of shares of the risky asset. The hedgers' trading volume and expected utility do not change with dual trading. Although the broker still offsets a portion of the hedgers' order flow, any impact of this on the hedgers' utility is exactly compensated by the reduction in market depth.[6]

Next, I analyze trading behavior when traders pay commission fees to the broker. As in Fishman and Longstaff (1992), the broker incurs a fixed and a variable cost of brokerage. Further, the brokerage business is competitive so that the broker's total income (expected trading profits plus expected commission income) net of brokerage costs, is zero. The commission fees are assumed fixed (i.e., independent of the order size) and paid irrespective of whether traders trade or not. This implies that traders never forgo a trade. Since the broker's commission income is independent of her customers' trading volume, the competitive commission fee is always lower with dual trading. The net profits of informed traders decrease and the net utility of uninformed traders increase with dual trading.[7]

In related literature, Roell (1990) has a broker observing the trades of uninformed traders. Her model does not include the effect of commission fees. Informed traders have higher profits when dual trading is banned.

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bid–ask spread or the depth (defined as the sensitivity of price to the order size). Smith and Whaley (1994), however, show a significant increase in the bid–ask spread as a result of a restriction on dual trading. Walsh and Dinehart (1991) also find some evidence that dual trading is associated with narrower bid–ask spreads.

[6] Specifically, the price impact of hedgers' trades is given by the product of the net uninformed order flow and the inverse of the market depth. Both the net uninformed order flow and the market depth are halved. Hence the result.

[7] The results when the broker's commission income is independent of the trading volume are consistent with Fishman and Longstaff (1992), where the trading volume is fixed at one share.

Uninformed traders whose trades are observed by the broker have higher profits with dual trading. Those uninformed traders whose trades are not observed by the broker are hurt by dual trading.

In Fishman and Longstaff (1992), the broker has private information about whether her customer is informed or not. Before commissions, all customers lose with dual trading. Including commissions, dual trading benefits uninformed traders and hurts informed traders. In contrast to this paper, Fishman and Longstaff (1992) assume that trading volume is fixed at one unit. As a result, the informed trader in their model fails to take into account the broker's mimicking behavior when formulating her optimal trading strategy. They also do not model the behavior of the customer when she is uninformed. On the other hand, they allow the customer and the dual trader to trade at different prices, and they also model the effect of frontrunning by the broker.

The remainder of this paper is organized as follows. Section I develops the basic dual trading model with noise traders, ignoring commission fees. In Section II, this is contrasted with a model where dual trading is prohibited. Results on the market impact of dual trading are obtained. Section III extends the model to allow for rational behavior by uninformed traders. Section IV studies the effect of dual trading on broker's fees. Section V concludes. All proofs are in the Appendix.

I. THE DUAL TRADING MODEL AND SOLUTION

A. *The Dual Trading Model*

Consider a market in which a single risky asset with unknown liquidation value v is traded. A group of m informed traders each receive signals s^i about the unknown value v prior to trading. The signals are of the form $s^i = v + e^i$, $i = 1, \ldots, m$, where the error terms e^i are independent of each other. In addition, a group of uninformed noise traders trade for liquidity reasons. Initially, the uninformed traders' motives for trading are not modeled. The basic model is subsequently extended to allow for rational behavior by uninformed traders.

Each informed trader $i = 1, \ldots, m$, submits a market order x_d^i to a broker. The noise traders also collectively submit market orders worth u to the same broker. The latter then places the total of the submitted orders $(x_d + u)$, where $x_d = \sum_{i=1}^{m} x_d^i$, to a marketmaker for execution.[8] In the dual trading model, the broker may also trade an amount d for her own ac-

[8] I employ the convention of labeling the decision variables of individual agents with a superscript and market variables with a subscript. The subscript d will refer to the solution in the dual trading model, and the subscript n to the nondual trading solution.

count. She may want to do so because, by observing the market orders x_{d}^{i} of the informed traders, she is able to infer some or all of their information s^{i}. In addition, it may be profitable for the broker to take the opposite position of the group of uninformed traders.[9] At this stage, I ignore broker's fees (which are introduced in Section IV).

It is assumed that, when the broker places her customer's order with the marketmaker, she simultaneously sends along her own order, d, as well. The marketmaker then fixes a single price p_{d} at which she will execute the total order flow, $y_{\mathrm{d}} = x_{\mathrm{d}} + d + u$. Following Kyle (1985), the marketmaker is assumed to be risk-neutral and competitive. Conditional on observing y_{d}, she earns zero expected profits.

The random variables in the model are v, u, e^{i}, $i = 1, \ldots, m$. The variables v and u are independent of each other and independent of e^{i}. All these variables are normally distributed with zero mean and finite variances Σ_v, Σ_u and Σ_e, respectively. Thus the m error terms are drawn from an identical distribution.[10] In addition, all investors follow linear trading rules $x_{\mathrm{d}}^{i} = A_{\mathrm{d}} s^{i}$, $i = 1, \ldots, m$ (for the informed), and $d = B_1 x_d + B_2 u$ (for the broker). This implies that the marketmaker's pricing rule is also linear: $p_{\mathrm{d}}(y_{\mathrm{d}}) = \Gamma_{\mathrm{d}} y_{\mathrm{d}}$, where $1/\Gamma_{\mathrm{d}}$ is the market depth parameter.

There are three distinct stages to this trading game:

(1) Informed traders receive their information and choose their desired trading volume. In making this decision, each informed trader is aware that, first, she is in competition with the other informed traders, and second, that the broker will piggyback on the information conveyed by her trading decision. Informed traders care about the broker's piggybacking because they receive a less favorable price for their trades as a consequence. Noise traders simply submit u.

(2) The broker observes u and x_{d}^{i} and infers that each informed trader has some information s^{i*}, $i = 1, \ldots, m$. Based on her inferences and u, she decides to trade an amount d.

(3) The marketmaker fixes a prices $p_{\mathrm{d}} = \Gamma_{\mathrm{d}} y_{\mathrm{d}}$, where $p_{\mathrm{d}} = E(v \mid y_{\mathrm{d}})$ and so $\Gamma_{\mathrm{d}} = \mathrm{cov}(v, y_{\mathrm{d}})/\mathrm{var}(y_{\mathrm{d}})$.

This suggests the following solution method. Fix Γ_{d} and suppose that each informed trader $i = 1, \ldots, m$, has decided to trade some amount x_{d}^{i} and uninformed traders have submitted market orders worth u. From each x_{d}^{i} the broker infers information s^{i*}. She then chooses d to maximize

[9] The broker is assumed to have no independent information regarding v. In a previous version of this paper (Sarkar, 1991), the broker has her own information but does not observe uninformed trades u. Also, $m = 1$. This makes for interesting interactions between the single insider and the broker. For example, for low precision of the broker's information, the insider's trade is decreasing in the precision of her own information!

[10] For $m = 2$, I have checked that the results are unchanged if informed traders have information of different precisions. I conjecture that this is true for finite m.

her expected profits, where the expectation is taken with respect to the vector $(s^{1*}, \ldots, s^{m*}, u)$. Each informed trader i then chooses x_d^i as a best response to $d(s^{1*}, \ldots, s^{m*}, u)$, the rival informed traders' decisions $x_d^j, j \neq i$, and uninformed trades u. Finally, Γ_d is obtained from the optimal trading rules and the marketmaker's zero profit assumption.

Because the equilibrium depends on the broker's beliefs, potentially there are multiple equilibria in the signaling game between informed traders and the broker. In this model, however, the fully separating outcome is the unique equilibrium.[11] In equilibrium, informed traders' information is fully revealed to the broker, so that $s^{i*} = s^i$, $i = 1, \ldots, m$.

B. *The Dual Trading Solution*

First, I solve the signaling game between informed traders and the broker. Given her observations of x_d^i and u, the broker chooses d to maximize her conditional expected profits given by $E(\pi | s^{1*}, \ldots, s^{m*}, u)$, where $\pi = (v - \Gamma_d y_d)\, d$. From the first-order condition, the optimal $d = [E(v|s^{1*}, \ldots, s^{m*}) - \Gamma_d(x_d + u)]/2\Gamma_d$. The second-order condition is satisfied by $\Gamma_d > 0$. Define $t = \Sigma_v/(\Sigma_v + \Sigma_e)$ and note that $0 \leq t \leq 1$ is a measure of the unconditional precision of s^i, $i = 1, \ldots, m$. For example, if $t = 1$ then s^i is a perfect signal. Then $E(v|s^{1*}, \ldots, s^{m*}) = ts^*/Q$, where $Q = [1 + t(m - 1)]$ and $s^* = \sum_{i=1}^m s^{i*}$. Therefore, the broker's optimal trade is

$$d = \frac{ts^*}{2Q\Gamma_d} - \frac{x_d + u}{2}. \tag{1}$$

In a separating equilibrium, $s^{i*} = s^i = x_d^i/A_d$ for each $i = 1, \ldots, m$. The presence of the dual trader thus has two opposing effects on an informed trader's incentive to trade. Suppose $x_d^i > 0$. If x_d^i increases, the broker infers that the ith informed trader's information is improved, and so s^* is higher as well. The broker trades more, d is higher, and so is the resulting price. Thus, this signaling effect tends to inhibit informed traders from trading aggressively.

On the other hand, a higher x_d also reduces d from the second term in (1). This is a "second-mover disadvantage" for the broker as she has to accommodate market orders of a given size by informed traders and tends to encourage informed trades. For finite m, however, the signaling effect always dominates the second-mover effect, so that $B_1 > 0$ in equilibrium. The broker optimally trades in the same direction as the net informed

[11] Uniqueness of the equilibrium follows because trading strategies and the pricing rule are linear and because the distribution of the signal is unbounded. As a result, there is a bijection between the range and the domain of each informed trader's trading strategy—which is a sufficient condition for uniqueness.

order flow. Also, the broker optimally trades in the opposite direction of the uninformed order flow.

Given (1), each informed trader i chooses x_d^i to maximize her conditional expected profits $E(I_d^i|s^i)$, where

$$I_d^i = \left(v - \Gamma_d\, d - \Gamma_d\, x_d^i - \Gamma_d \sum_{j\neq i} x_d^j - \Gamma_d u\right) x_d^i.$$

After incorporating the optimal value of d from Eq. (1) into I_d^i, the first-order condition for x_d^i, $i \neq j$, is

$$\frac{t(1 + Q)s^i}{2} = \Gamma_d[x_d^i + 0.5(m - 1)E(x_d^j|s^i)] + \frac{ts^{i*}}{Q}. \tag{2}$$

Equation (2) says that the marginal value of an additional trade for the ith informed trader is equal to its marginal cost. This cost has two components: the change in the price due to her own and her rivals' expected trades plus the change in the broker's inference regarding her information. After using the facts that (i) $s^{i*} = s^i = x_d^i/A_d$ in equilibrium and (ii) $E(s^j|s^i) = ts^i$ for $j \neq i$, A_d is obtained as the coefficient of s^i in (2):

$$A_d = \frac{t^2(m - 1)}{\Gamma_d Q(Q + 1)}. \tag{3}$$

From (3), $A_d = 0$ when $m = 1$. But $A_d = 0$ cannot be a separating equilibrium since the functions $x_d^i = A_d s^i$, $i = 1, \ldots, m$, are then no longer invertible.

LEMMA 1. *When $m = 1$, there is no solution to the dual trading model.*

The result can be interpreted as follows. The inhibiting effect of the broker's piggybacking behavior on any individual informed trader is inversely related to m, the number of informed customers the broker has. For $m = 1$, this inhibiting effect exactly offsets the marginal value of an extra trade for the individual informed trader as the first-order condition (2) reduces to

$$t(s^i - s^{i*}) = \Gamma_d x_d^i. \tag{4}$$

So, for any $x_d^i > 0$, the marginal cost of an additional trade for the informed trader always exceeds its marginal benefits.[12] Substituting (3) into

[12] In Sarkar (1991), an equilibrium exists even with $m = 1$ so long as the precision of the broker's information is positive. The reason is that, if the broker has an independent source of information about v, she attaches relatively less weight to her observation of the informed trade. The change in the broker's inference, when the informed trader buys or sells an extra share, no longer fully offsets the marginal value of that extra share traded.

(1), the optimal dual trading function is given by

$$d = \frac{x_d}{t(m-1)} - \frac{u}{2}. \tag{5}$$

Finally, by using (3) and (5) in conjunction with the marketmaker's zero profit assumption, the optimal value of market depth is derived as

$$\Gamma_d = 2\frac{\sqrt{mt\Sigma_v}}{(1+Q)\sqrt{\Sigma_u}}. \tag{6}$$

Proposition 1 fully characterizes the dual trading equilibrium.

PROPOSITION 1. *If $m > 1$ and $t > 0$, there exists a unique solution to the dual trading model in which $x_d^i = A_d s^i$, $i = 1, \ldots, m$, $d = B_1 x_d - u/2)$ and $p_d = \Gamma_d y_d$, where A_d is given by (3), B_1 by the coefficient of x_d in (5), and Γ_d by (6).*

II. THE MARKET IMPACT OF DUAL TRADING

In weighing the costs and benefits of dual trading, a regulator might be interested in its effect on total trading volume and profits, market depth, and price efficiency. These market effects are now studied.

A. *The Nondual Trading Model*

I will compare the dual trading solution obtained in Section I with the solution obtained when dual trading is banned. The broker then merely brings her customers' orders to the market. The resulting trading game is a Cournot–Nash game in trading quantities. Each informed trader places an order x_n^i with the broker based on her information s^i. The broker submits the total order flow $y_n = x_n + u$ (where x_n is the net informed order flow in the nondual trading market) to the marketmaker for execution. The price is $p_n = \Gamma_n y_n$. Lemma 2 describes the nondual trading equilibrium.

LEMMA 2. *If there is no dual trading, a solution always exists provided that $t > 0$. Each informed trader $i = 1, \ldots, m$ trades $x_n^i = A_n s^i$ and the price is $p_n = \Gamma_n y_n$, where*

$$A_n = \frac{t}{\Gamma_n(1+Q)}, \qquad \Gamma_n = \frac{\sqrt{mt\Sigma_v}}{(1+Q)\sqrt{\Sigma_u}}. \tag{7}$$

Proof. See the proof of Lemma 1 in Admati and Pfleiderer (1988).

B. *Trading Volume and Gross Profits, Market Depth, and Price Efficiency*

Due to piggybacking by the broker, it is reasonable to expect that $A_d < A_n$. The difference in informed trading intensities A_i, i = d, n, depends upon the market depths Γ_d and Γ_n. By inspection of (6) and (7), $\Gamma_d = 2\Gamma_n$: relative to the market without dual trading, market depth is half its value with dual trading. This reflects the fact that the dual trading broker offsets half of the uninformed order flow. Given this result, the difference in informed trading intensities is

$$A_n - A_d = \frac{t}{\Gamma_d Q}, \tag{8}$$

which is positive for $t > 0$. The term t/Q represents what the broker learns about the unknown v from observing the m vector of informed market orders. The more informative is this observation, the greater is the shrinkage of the informed order flow in the dual trading market. The difference $(A_n - A_d)$ is also positively related to market depth, since a deeper market allows the broker to trade larger amounts with less concern about the price impact.

However, the broker herself provides an additional source of trading activity in the dual trading market. Considering the dual trading and informed trading activities together:

$$x_d + d = \frac{1}{\Gamma_d}\frac{ts}{1 + Q} - \frac{u}{2} \tag{9}$$

$$x_n = \frac{1}{\Gamma_n}\frac{ts}{1 + Q}. \tag{10}$$

From (9) and (10), differences in the net order flows between the two markets occur because the broker offsets part of the noise trading order flow. As a result, total informed order flow (by the informed traders *and* the broker) is halved due to the halving of market depth, and, further, the net uninformed order flow (of the broker plus the noise traders) is also halved. Hence, $y_n = 2y_d$: the net order flow is twice as high without dual trading.

Define the expected uninformed trading volume as $E(|u|) = \sqrt{\Sigma_u}/\sqrt{2\pi}$. Define the expected net informed trading volume in market i = d, n as $E(|x_i|) = A_i\sqrt{m\Sigma_s}/\sqrt{2\pi}$. Considering noise trades and the broker's portion of the uninformed trades together, the expected uninformed trading volume is higher by an amount $\sqrt{\Sigma_u}/2\sqrt{2\pi}$ with dual trading. The expected informed trading volume (of the informed traders and the broker together) is lower with dual trading. These two effects are exactly offsetting so that the expected total trading volume is unchanged with dual trading.

Define price efficiency PI_i, $i = \text{d, n}$, as $\Sigma_v - \text{var}(v|p_i)$. It can be shown that $\text{PI}_i = (\Gamma_i)^2\Sigma_{y_i}$, where Σ_{y_i} is the variance of the net order flow in market $i = \text{d, n}$. PI_i depends positively on the volatility of the net order flow (since this depends positively on the informed trading intensities A_i) and inversely on market depth (since prices are less sensitive to volume, and hence to information, in deep markets). It follows that $\text{PI}_\text{d} = \text{PI}_\text{n}$ because, relative to the nondual trading market, the reduction in the variance of the net order flow is exactly offset by the reduction in market depth in the dual trading regime.

Let I_i denote the combined unconditional expected profits of the informed group (before observing any signals or paying any commissions) in market i, $i = \text{d, n}$. Since the price level is defined as $p_i = \Gamma_i y_i$, the price levels in the two markets are equal. It follows that $I_\text{d} < I_\text{n}$: gross profits of informed traders are strictly lower with dual trading (since the price level is the same but the informed order flow is lower). Total trading profits in the dual trading market is $I_\text{d} + W$, where W is the broker's unconditional expected trading profits. Total *informed* trading profits in the dual trading market (I_d plus that part of the broker's profits obtained from mimicking informed trades) is half of I_n (since the price level is the same but the total informed order flow is half of x_n). But the broker also profits from her observance of noise trading, and the amount of this profit equals $\Gamma_\text{d}\Sigma_u/4$ or $\Gamma_\text{n}\Sigma_u/2$, which is exactly half of I_n. Thus $I_\text{d} + W = I_\text{n}$.

In equilibrium, uninformed traders as a group suffer losses, and the amount of their losses mirrors the total trading profits of informed traders and the broker. Denote $L_\text{g} = I_\text{d} - I_\text{n} + W$ as the difference in the gross losses of uninformed traders between the dual and nondual trading regimes. Therefore, $L_\text{g} = 0$.

PROPOSITION 2. (1) $y_\text{n} = 2y_\text{d}$ and $\Gamma_\text{d} = 2\Gamma_\text{n}$. *The net order flow and market depth in the dual trading market are half their values in the nondual trading market.* (2) *The expected total trading volume and price efficiency are the same with or without dual trading. The combined gross trading profits of informed traders plus the dual trader in the dual trading solution are equal to the gross trading profits of informed traders in the nondual trading solution.* (3) $I_\text{d} < I_\text{n}$ *and* $L_\text{g} = 0$. *Gross profits of informed traders are lower with dual trading. Gross losses of noise traders are unchanged.*

In the following section, the basic model is extended to allow for rational behavior by uninformed traders.

III. HEDGING BY UNINFORMED TRADERS

There are h risk-averse uninformed traders ("hedgers") who trade exclusively for risk-sharing purposes. The model follows Spiegel and Subrahmanyam (1992). Each hedger j has random endowment w^j, which is assumed to be normally distributed with mean zero and variance Σ_w. The $w^j, j = 1, \ldots, h$, are independent of each other and all other random variables in the model. All hedgers have negative exponential utility functions with risk-aversion parameter R.

Suppose that in market i all hedgers $j = 1, \ldots, h$ submit market orders u_i^j to the broker and follow linear trading rules of the form $u_i^j = D_i w^j$. Let the total uninformed trading volume in market i be $u_i = \Sigma_{j=1}^h u_i^j$. If π_i^j is the profit of the jth hedger in the ith market, then u_i^j is chosen to maximize her utility or certainty-equivalent profits $G_i^j = E(\pi_i^j|w^j) - (R/2)\,\mathrm{var}(\pi_i^j|w^j)$. Let G_i, i = d, n, be the sum of the utilities of all h hedgers in the ith market. The informed traders and the broker's maximization problems remains the same as before, since each w^j is independent of v.[13] Market depth is now positively related to the magnitude of the "hedge factor" D_i (since this increases the variance of the total order flow) and to the risk-aversion parameter R. Further, the equilibrium $D_i < 0$ since the marginal utility of the hedgers from a purchase (sale) is negative if endowments are positive (negative). Lemma 3 describes the equilibrium for the nondual trading market. The Appendix describes the dual trading equilibrium.

LEMMA 3. *An equilibrium to the hedger model exists if R satisfies*

$$R\Sigma_v(2 - t) > 2\sqrt{mt\Sigma_v}/\sqrt{h\Sigma_w}. \tag{11}$$

In equilibrium, each hedger $j = 1, \ldots, h$ *trades* $u_n^j = D_n w^j$, *where* $D_n < 0$, *market depth is* $1/\Gamma_n$, *and*

$$-D_n\Gamma_n = \frac{\sqrt{mt\Sigma_v}}{(1 + Q)\sqrt{h\Sigma_w}}, \tag{12}$$

where

$$D_n = \frac{2\sqrt{mt\Sigma_v/h\Sigma_w} - R\Sigma_v(2 - t)}{R\Sigma_v(2 - t) - R\Sigma_v mt/h(1 + Q)}. \tag{13}$$

[13] The actual informed and dual trading volumes will be different since market depth will be different in general.

Proof. See the proof of Proposition 1 in Spiegel and Subrahmanyam (1992).

From (12) and (A13) in the Appendix, $\Gamma_d ab(D_d) = 2\Gamma_n ab(D_n)$, where $ab(D_i)$ denotes the absolute value of D_i. How does dual trading affect the choice of D_i? The price impact of uninformed order flows is given by $\Gamma_i u_i$. Conjecture that $\Gamma_d = 2\Gamma_n$. Then, since net uninformed order flows (including those by the broker) are halved with dual trading, the price impact of uninformed trades is identical in both markets, and so $D_d = D_n$. This in turn means that $u_d = u_n$, and the initial conjecture regarding the market depths must be correct as well.[14] As before, the price levels are identical in the two markets, and, by implication, $G_d = G_n$. Hence, the results on the market impact of dual trading of the previous section are robust to the introduction of rational uninformed traders.

PROPOSITION 3: $D_d = D_n$, $G_d = G_n$, *and* $\Gamma_d = 2\Gamma_n$. *The hedgers' trading volumes and gross utilities are the same with or without dual trading. The depth of the market is halved when dual trading is permitted.*

IV. TRADING WITH BROKER'S FEES

Suppose that, in market i, the broker charges a fee of $\$c_i$. Traders make their decisions in the following sequence. At stage zero, the broker determines the fee c_i. At stage one, informed traders observe their signals and c_i. Uninformed traders observe their endowments and c_i. Then both groups decide how much to trade. At stage two, if dual trading is allowed, the broker's trade size is also contingent on c_d and her observance of informed and uninformed trades. Finally, a price is set in the manner specified earlier.

The broker faces a fixed cost k_0 and a variable cost k_1 of conducting business, both nonnegative. Following Fishman and Longstaff (1992), I assume that the brokerage business is competitive, and so c_i is chosen so that the broker's expected trading profits plus expected fee income equals her total costs of brokerage.

Each of the $(m + h)$ traders must pay a fixed (i.e., independent of the order size) fee. The fee is paid irrespective of whether the trader makes a trade; think of this charge as a market access fee. The variable cost is k_1 per trade. In this case, the fee does not affect traders' decisions regarding how much and whether to trade.

When the jth informed trader trades, her net profit in market i is $N_i^j =$

[14] Note that this is precisely the situation considered in the previous section where u was exogenously fixed at the same level in the two markets. This result is proved in the Appendix.

$\Pi_i^j - c_i$. If she does not trade, her loss is $-c_i$. So, she will always trade. Also, her equilibrium order size and gross profits will not change since the first-order conditions for her trading decision are unaffected. Similar remarks apply to uninformed traders. Thus, the total number of traders ($m + h$) is also the number of trades occurring in equilibrium. The broker's fees are determined by the following equations:

$$c_d = k_1 + \frac{k_0 - W}{m + h} \tag{14}$$

$$c_n = k_1 + \frac{k_0}{m + h}. \tag{15}$$

Expressions (14) and (15) equate the profit margin per trade ($c_i - k_1$) in market i to the fixed cost of brokerage per trade, net of the broker's profits. Since the broker's dual trading profits are positive, $c_d < c_n$. Further, the difference in broker's fees ($c_n - c_d$) is equal to the dual trading profits per trade. For the jth uninformed trader, her net profits are $\pi_d^j - c_d$ in the dual trading market and $\pi_n^j - c_n$ in the nondual market. From Proposition 3, her gross profits (and utility) are unchanged with dual trading. Since $c_d < c_n$, her net profit (and utility) increases with dual trading. For informed traders, Proposition 2 says that gross profits decrease by W, the amount of dual trading profits. On the other hand, the reduction in broker's fees saves them $[m/(m + h)]W$. Thus, the net profits of informed traders decrease with dual trading.

PROPOSITION 4: *Suppose that in market i = d, n, traders pay a broker's fee c_i which is independent of the order size and is paid irrespective of whether they trade or not. Then* (1) $c_d < c_n$ *and* (2) *net of c_i, the utility of uninformed traders higher and profits of informed traders are lower with dual trading.*

V. CONCLUSION

This paper considers the effects of allowing a broker to trade on her own account (to dual trade) in addition to executing customers' orders. In the model without commission fees, dual trading leads to a reduction in the net order flow and the market depth, but expected trading volume and price efficiency are unaffected. Informed traders have lower profits with dual trading because they are forced to be less aggressive in anticipation of the broker mimicking their trades and piggybacking on their information. The broker also trades in the opposite direction of the uninformed order flow. Since the net uninformed order flow and the market depth both decrease by the same proportion, however, the price impact of an uninformed trade and the variance of uninformed profits are unchanged. So, uninformed traders neither gain nor lose with dual trading.

With broker's fees fixed, as in the case of access fees, the competitive fee is lower, and, net of the fee, informed traders earn lower profits, and uninformed traders enjoy higher utility with dual trading.

In Locke *et al.* (1994), some dual traders appear more likely than others to profit from their customers' order flow. Moreover, informed customers seem to avoid dual traders who are more liable to exploit their information and, instead, favor dual traders who are mainly brokers. This matching of customers and brokers according to their specific needs is a subject for future research.

APPENDIX

Proof of Lemma 1. Let $E(v|s^{i*}, \ldots, s^{m*}) = as^*$, where $s^* = \sum_{i=1}^{m} s^{i*}$. Applying Bayes' rule yields $a = (1/\Sigma_e)/(1/\Sigma_v + m/\Sigma_e) = t/(1 + (m - 1)t)$, where $t = \Sigma_v/(\Sigma_v + \Sigma_e)$. This gives the optimal dual trade $d(s^*, x_d)$ as given in (1).

Incorporating (1), each informed trader i's profits I_d^i can be written as

$$I_d^i = \left(v - \frac{t}{1 + (m - 1)t}\frac{s^*}{2} - \frac{\Gamma_d}{2}x_d - \frac{\Gamma_d u}{2}\right) x_d^i. \tag{A1}$$

Substituting $s^{i*} = s^i = x_d^i/A_d$ for each $i = 1, \ldots, m$ into (A1) results in the expression

$$\begin{aligned} E(I_d^i|s^i) = \Bigg[ts^i &- \frac{x_d^i}{2}\left(\Gamma_d + \frac{t}{1 + (m - 1)t}\frac{1}{A_d}\right. \\ &\left. - \sum_{j \neq i} \frac{E(x_d^j|s^i)}{2}\left(\Gamma_d + \frac{1}{A_d}\frac{t}{1 + (m - 1)t}\right)\right] x_d^i. \end{aligned} \tag{A2}$$

Substituting $E(x_d^j|s^i) = A_d t s^i$ for each $j \neq i$ into (A2) and then differentiating with respect to x_d^i gives (2). When $m = 1$, (2) has the form

$$x_d^i\left(\frac{t}{A_d} + \Gamma_d\right) = ts^i. \tag{A3}$$

It is easily checked that there is no $A_d > 0$ such that $x_d^i = A_d s^i$ has a solution. ■

Proof of Proposition 1. From (3) and (5), $y_d = x_d + d + u = (x_d\,[1 + t(m - 1)])/[t(m - 1)] + u/2)$ or

$$y_d = \frac{ts}{[2 + t(m - 1)]} \cdot \frac{1}{\Gamma_d} + \frac{u}{2}, \qquad \text{where } s = \sum_{i=1}^{m} s^i. \tag{A4}$$

Equation (6) follows from solving $\Gamma_d = \text{cov}(v, y_d)/\text{var}(y_d)$. ■

Proof of Proposition 2. Price informativeness PI_i, $i = \text{d, n}$, is defined as

$$\text{PI}_i = \Sigma_v - \text{var}(v|p_i) = \Gamma_i^2 \Sigma_{y_i}. \tag{A5}$$

Since $\Gamma_d = 2\Gamma_n$ but $y_d = y_n/2$, we have $\text{PI}_d = \text{PI}_n$.

Now, $I_i = E([v - p_i]x_i)$ is the total unconditional expected profits of informed traders in market $i = \text{d, n}$. Note that $I_d < I_n$ since $p_d = p_n$ and $x_d < x_n$.

$$\begin{aligned} I_d + W &= E([v - p_d]\,[x_d + d]) \\ &= E\left\{[v - p_n]\left[\frac{x_n}{2} - \frac{u}{2}\right]\right\} = \frac{I_n}{2} + \frac{\Gamma_n \Sigma_u}{2}. \end{aligned} \tag{A6}$$

Since $\Gamma_n \Sigma_u = \sqrt{mt\Sigma_v\Sigma_u}/(1 + Q) = I_n$, it follows that $I_d + W = I_n$.

The expected trading volume in the nondual trading market is $(A_n \sqrt{m\Sigma_s})/\sqrt{2\pi} + \sqrt{\Sigma_u}/\sqrt{2\pi}$. The expected trading volume in the dual trading market is $QA_d\sqrt{m\Sigma_s}/\sqrt{2\pi}\ (Q - 1) + \frac{3}{2}\sqrt{\Sigma_u}/\sqrt{2\pi}$. Substituting for the values of A_d and A_n, we get the result that the expected trading volumes are the same. ■

Proof of Proposition 3. In the dual trading market, the profits of the jth hedger are

$$\pi_d^j = v(u_d^j + w^j) - \Gamma_d u_d^j \left(u_d^j + D_d \sum_{m \neq j} w^m + x_d + d\right). \tag{A7}$$

Since $x_d + d = ts/\Gamma_d(1 + Q) - u_d/2$, this expression becomes

$$\pi_d^j = v(u_d^j + w^j) - \frac{\Gamma_d u_d^j}{2}\left(u_d^j + D_d \sum_{m \neq j} w^m\right) - \frac{ts}{1 + Q} u_d^j, \tag{A8}$$

$$E(\pi_d^j|w^j) = -\frac{\Gamma_d}{2}(u_d^j)^2, \tag{A9}$$

$$\begin{aligned} var(\pi_d^j|w^j) &= \Sigma_v(w^j)^2 + (u_d^j)^2\left[\Sigma_v\left(1 - \frac{mt(2 + Q)}{(1 + Q)^2}\right)\right. \\ &\quad \left. + \frac{(\Gamma_d D_d)^2}{4}(h - 1)\,\Sigma_w\right] + 2\Sigma_v w^j u_d^j \frac{(2 - t)}{1 + Q}. \end{aligned} \tag{A10}$$

The certainty-equivalent profits $G_{\mathrm{d}}^{j} = E(\pi_{\mathrm{d}}^{j}|w^{j}) - (R/2 \operatorname{var}(\pi_{\mathrm{d}}^{j}|w^{j})$. Differentiating G_{d}^{j} with respect to u_{d}^{j},

$$-\Gamma_{\mathrm{d}}u^{j}_{\mathrm{d}} - Ru^{j}_{\mathrm{d}}\left[\Sigma_{v}\left(1 - \frac{mt(2+Q)}{(1+Q)^{2}}\right) + \frac{(\Gamma_{\mathrm{d}}D_{\mathrm{d}})^{2}}{4}(h-1)\,\Sigma_{w}\right] - R\Sigma_{v}w^{j}\frac{(2-t)}{1+Q} = 0. \tag{A11}$$

Equating D_{d} to the coefficient of w^{j} in (A11) and solving,

$$RD_{\mathrm{d}}\frac{(\Gamma_{d}D_{d})^{2}}{4}(h-1)\,\Sigma_{w} + D_{\mathrm{d}}\left[\Gamma_{\mathrm{d}} + R\Sigma_{v}\left(1 - \frac{mt(Q+2)}{(1+Q)^{2}}\right)\right] + \frac{R\Sigma_{v}(2-t)}{1+Q} = 0. \tag{A12}$$

Solving for Γ_{d},

$$\Gamma_{\mathrm{d}}D_{\mathrm{d}} = \frac{-2\sqrt{mt\Sigma_{v}}}{(1+Q)\sqrt{h\Sigma_{w}}}. \tag{A13}$$

From (12) and (A13), $\Gamma_{\mathrm{d}}D_{\mathrm{d}} = 2\Gamma_{\mathrm{n}}D_{\mathrm{n}}$. Solving for D_{d} shows that $D_{\mathrm{d}} = D_{\mathrm{n}}$. It follows that $G_{\mathrm{n}}^{j} = G_{\mathrm{d}}^{j}$ for each j. Thus, $G_{\mathrm{d}} = G_{\mathrm{n}}$. Finally, $D_{\mathrm{d}} = D_{n}$ implies that $\Gamma_{\mathrm{d}} = 2\Gamma_{\mathrm{n}}$. ■

Proof of Proposition 4. From (14) and (15), $c_{\mathrm{n}} - c_{\mathrm{d}} = W/(m+h)$. The total benefit to the broker's $(m+h)$ customers is $(m+h)(c_{\mathrm{n}} - c_{\mathrm{d}}) = W$. The share of the m informed traders as $[m/(m+h)]\,W < W$, which is their loss from dual trading. ■

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